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THE *MENO* AND THE SECOND PROBLEM OF GEOMETRY AT 86e[1]

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The aim of this paper is two-fold: firstly, to argue for the claim that the two problems of geometry presented in the Meno *seem to be connected to each other, and secondly, to offer, in connection with the first claim, a conjecture concerning the nature of the second problem of geometry brought up in the dialogue at 86e. This paper offers, in particular, a historical reconstruction of how we should understand this problem of construction in geometry.*

INTRODUCTION

Plato's dialogue, the *Meno*, is such a rich text philosophically that it raises and hosts a significant number of important problems in philosophy. Let me begin by citing the moderately straightforward translation by Thomas L. Heath (1921, 299) of the much-studied part of the dialogue at 86e:

> ...when they (geometers) are asked, for example, as regards a given area, whether it is possible for this area to be inscribed in the form of a triangle in a given circle. The answer might be, "I do not yet know whether this area is such as can be so inscribed, but I think I can suggest a hypothesis which will be useful for the purpose; I mean the following. If the given area is such as, when one has applied it (as a rectangle) to the given straight line in the circle…, it is deficient by a figure (rectangle) similar to the very figure which is applied, then one alternative seems to me to result, while again another results if it is impossible for what I said to be done with it. Accordingly, by using a hypothesis, I am ready to tell you what results with regard to the inscribing of the figure in the circle, namely, whether the problem is possible or impossible.

Another rendering of the passage given above is also proposed by Wilbur Richard Knorr (1986, 71) in his book, *The ancient tradition of geometric problems*:

> I say "from hypothesis" in the manner that the geometers often make inquiry, whenever someone has asked them, for instance about an area,

> whether this area here can be stretched out as a triangle in this circle here, one would say "I don't yet know whether this is of such a sort, but I think that as a certain hypothesis the following will assist in the matter. If this area is such that the one who has stretched (it) along its given line (makes it) fall short by an area such as is the stretched (area) itself, then it seems to me that a certain result follows, but a different result, if it is impossible that these things be done. Having hypothesized, then, I wish to say to you whether the result about its stretching in the circle is impossible or not.

There are certain questions arising from this passage, and these questions can be considered in three groups:

(1) The passage has a troublesome nature. It contains some linguistic ambiguities and obscurities. The geometric problem addressed there is not well stated; so, it is left ambiguous. For example, Geoffrey E. R. Lloyd (2004, 179) states that the example is "unnecessarily obscure." The issue of the clarity of the passage is likewise addressed by, for example, Knorr (1986, 71-72). It is unanimously agreed that it is not clear what we are dealing with in the passage. For example, what is meant by the term *area* (*xorion*) appearing in the passage? Is it a *square, rectangle,* or any *rectilinear area/figure*? Again, is it the *diameter* or a *chord* of the circle that is meant by *the given straight line* (*of the circle*)? And, lastly, how should we take these figures? S*imilar* in geometrical sense or *equal*? So the general question is how one should interpret the passage.

(2)The geometrical problem is not only vaguely formulated, but as Heath (1921, 298) points out, it is a difficult problem of geometry, too. Note that Plato introduces this problem into the dialogue in order to illustrate *the method of hypothesis*. There is nothing in the *Meno* to suggest why Plato chooses such a difficult problem just to illuminate the method in question. So, no reason is forthcoming in the dialogue for this. Could Plato not have chosen a much easier example just to make clear the so-called method of hypothesis?

(3) In connection with the difficulty mentioned above in (2), there is also a question about the manner in which the problem was brought about in the dialogue. Why is it introduced in such a way as if there was no need to elaborate it further? Why is it stated as if each of the discussants knew what the problem was all about, and as if it was a well-known problem of geometry?

Furthermore, two issues arise concerning the relevance of the second geometrical problem to the first geometrical problem studied in the dialogue at 82b, and to the whole dialogue: Is the problem in question in any way relevant or instrumental to the understanding of the whole dialogue? And what is the connection between the two geometrical problems presented in the *Meno*, if there is any? Since these are the issues to be addressed in this paper, I would just like to draw our attention to them at this point.

With respect to the first issue, there are some who claim that identifying the exact nature of the problem is dispensable, and that what matters in the passage is to see how the method of hypothesis works; namely, when a geometer faces a problem, s/he approaches it by maintaining that if certain conditions are satisfied, a particular result will follow; if not, another solution will result . It is maintained that the gist of the passage for philosophers is to understand what this method is all about and how it works and applies. So, identifying the problem and overcoming the ambiguities in the passage have no bearing whatsoever to the understanding of the whole dialogue. Nevertheless, it might be amusing to consider and study the passage and the problem though (see, e.g., Taylor 1955, 138-39, n.3).

This problem has been, as John E. Thomas (1980, 166) states, "a perennial source of puzzlement to commentators" and Heath (1921, 298) mentions that "C. Blass, writing in 1861, knew thirty different interpretations" of the second problem. It seems that the question of what the precise nature of the problem could be has, as Richard S. Bluck (1961, 322) points out, "exercised the ingenuity of numerous scholars, but no completely satisfactory solution has been found." So it is not surprising that there are many interpretations of what the actual problem could amount to. I, too, would like to offer a conjecture concerning what this problem could be. But before I do so, I would like to outline briefly their solutions, but without discussing their principal strengths and weaknesses.

Among those who have so far tackled this issue because of either puzzlement or some other reasons are Adolph Benecke (1867), Samuel H. Butcher (1888), John Cook Wilson (1903), Heath (1921), Arthur S. L. Farquharson (1923), A. Heijboer (1955), Bluck (1961), Thomas (1980) and Lloyd (2004).

The solutions they have offered can mainly be treated in three groups:

(a) *Benecke's interpretation* (see Bluck 1961, 447f.; Heath 1921, 302; Lloyd 2004, 171): Benecke in his 1867 book proposes that the figure to be inscribed as a triangle into a given circle is a *square*; for the square in question is already set out in the discussion with the slave boy at 82b. So, the square is not any, but that of side two feet.

Moreover, as for the *straight line of the circle* upon which the square is to be applied, Benecke takes it to be the *diameter* of the circle. The figure, when applied on the diameter of the circle which would fall short by an area, is not just *similar* but *equal* to the figure we started with, that is, the square.

So the problem then for Benecke is: can an isosceles right-angled triangle, being equal to the aforementioned square, be inscribed in a given circle? As Heath (1921, 302-303) rightly observes, this is possible only if the radius of the circle is two feet in length. So the construction problem would be represented as the one in Figure 1.

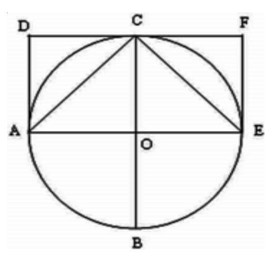

Fig. 1

If AE and CB are two diameters cutting each other at right angle at O, then the triangle to be inscribed is ACE. The square ADCO, which is formed by radii OA and OC, is the rectangle applied, and the rectangle by which it falls short is also a square, OEFC, and equal to the one applied ADCO.

(b) *Butcher's interpretation* (see Bluck 1961, 442f.; Heath 1921, 299-300; Lloyd 2004, 171-73; and Knorr 1986, 72 and 92, n. 55]: In Butcher's interpretation, given in his

1888 paper, the area to be inscribed is taken, as opposed to Benecke's rendering, not a *square*, but a *rectangle*. This rectangle is to be applied on *the diameter of the circle*, as in Benecke's interpretation. So, when it is applied, it will be short by an area, which is not *identical* or *equal*, but *similar* to the figure initially given, that is, the rectangle itself.

So, according to Butcher, the problem is whether a given rectangle can be inscribed as a triangle in a given circle. The problem of construction would then look like the one in Figure 2.

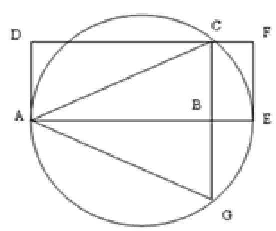

Fig. 2

If this is the case, then one of the corner points of the rectangle, ABCD, which is applied, must lie on the circumference of the circle. In Figure 2, if the rectangles ABCD and BEFC are similar, then AB:BC= BE:BC, so that BC^2=AB.BE. Then it will be seen that the point C lies on the circumference of the circle of which AE is the diameter. If BC is extended to G so that BC would be equal to BG, G will also lie on the circumference of the circle. Then the triangle ACG constructed can be seen to be inscribable in the given circle, and is equal in area to the rectangle ABCD.

So, if the condition as understood in the way outlined above holds, then the given rectangle can be inscribed as a triangle in the given circle. However, if the condition does not hold, then it *may* or *may not* be possible to inscribe it as required. What we need here is the condition, as stated by Bluck (1961, 444), that "the given area should not be greater than the greatest possible triangle which can be inscribed in the given circle." So, if the condition is not fulfilled, we will then not be in a position to draw any conclusion.

(c) *Wilson's interpretation* (see Bluck 1961, 445; Heath 1921, 298ff.; Lloyd 2004, 173-74; and Knorr 1986, 72-73): Wilson's interpretation, given in his 1903 paper, is adopted and adapted by both Heath (1921, 298ff.) and Knorr (1986, 72ff.). According to Wilson, the term *xorion* may be taken in Butcher's sense, that is, a rectangle; but he (1903, 230) says that "there seems to be no instance of *xorion* standing really by itself for a rectangle" in either Euclid or Pappus. Wilson (1903, 226) thinks that it would be better to take its proper meaning as "a figure from the point of view of its area, not of its shape." So, in this interpretation, the area to be inscribed is not thought of as a certain geometrical figure or shape, but of being any rectilinear area in order to avoid any objection. It is a general understanding of the term as area at 86e.

This area is to be applied on the diameter of the circle as in the earlier interpretations. The construction problem is then how to inscribe this given area in the form of a triangle, which falls short by an area geometrically similar to itself.

Let the given area be X and ACG be an isosceles triangle being equal in area to X, which is to be inscribed in the given circle. Moreover, let AE be the diameter of the circle bisecting CG in B at right angle. One can then construct the following rectangles ABCD and BEFC. Since the triangles ADC, ABC, and ABG are congruent to each other, ABCD will be equal in area to X. This figure, when applied to the diameter AE of the circle, falls short by a rectilinear area, or in this case, the rectangle BEFC (see Figure 3).

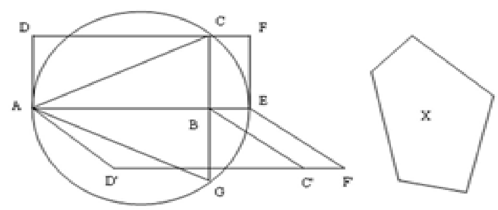

Fig. 3

It is also the case that because of AB.BE=BC^2, then AB:BC=BC:BE. It is easy to see that the rectangle BEFC is similar to ABCD. Then if the isosceles triangle, AGC, can be inscribed in the given circle being equal in area to the given rectilinear figure/area, it will be possible to construct a rectangle being equal to the given rectilinear figure/area.

However, in order for this to hold, point C must lie on the circumference of the circle. This is a crucial difference between Butcher's and Wilson's interpretations; namely, in Butcher's case the rectangle ABCD is taken as given whereas on Wilson's account, the point C lying on the circumference of the circle has to be constructed.

Having provided the general outline of the proposed solutions to the problem given at 86e in the *Meno*, let me now go back to the questions posed in (1)-(3) at the beginning of the paper, and consider them; for I think that these questions are instrumental, and have heuristic value in shaping the reconstruction I shall propose concerning the geometrical problem at 86e in the *Meno*.

Issue of Vagueness

Now, I would like to consider the questions in (1), that is, the issues of vagueness and the ambiguous character of the formulation of the problem, because of which the question of how to render the passage arises. What I would like to do here is to ask the following questions, rather than the question of how to render the passage: Is this very feature of the formulation something that one can blame Plato for his incompetence in geometry? Or is it something that stems from the very nature of the geometric problem itself? In other words, is this because of the state of geometry at the time? Lloyd (2004, 176-78) asks similar questions, too.

One can go for the case that Plato was not in command of the geometrical knowledge of his time. However, if this is the case, then as Lloyd (2004, 177) points out, there is not much to say. But the following remarks are in order: it is not a similar picture we get from the discussion of the first geometrical example in the *Meno*. There Plato does seem to be perfectly in command of geometrical knowledge then available. However, one may argue against this by maintaining that the first example is a simple one so that it cannot be taken as a sign of Plato's competence in geometry. Although one may see it today as an easy and a well defined problem of construction in geometry, this might very well not be the case when the problem was brought about at the first time or in its early days.

So, we do not seem to have had any conclusive evidence in favour of the claim that Plato was not in command of geometry at the time. Since arguing in favour of or against the claim would take us away from the main point I would like to make here, I shall assume that Plato was reliable concerning the geometrical knowledge at the time, though he was not certainly a creative mathematician.

One can still argue by maintaining that although Plato could be in command of geometry of his time, he might have been responsible for the poor formulation of the problem; for he is well known for his enthusiasm for mathematical obscurities. At this point what I would like to point out is that if this is the case, then one expects to see more or less a similar kind of attitude in the discussion of the first geometrical problem in the *Meno*. However, this is not what we observe there. I think that the belief in the claim presented above stems from not a particular passion Plato has for obscurities, but from the fact that Plato was keen for problems and studies in geometry for the reason of obtaining some lessons from and making points of method and modifying procedures already familiar among geometers at the time. As Knorr (1986, 28) rightly observes, this "certainly typifies the later philosophical tradition, as represented by Plato, Aristotle, Stoics, and the Sceptics, for instance, and their stance toward the technical sciences of their times." So, if any obscurity exists in the problems and studies they are dealing with, it must come from the material itself, not from a specific passion of individuals like Plato himself for such obscurities. Any presence of such obscurity in the materials would seem to be very natural because it was the first time these problems appeared, and were dealt with.

Choice of a Difficult Geometrical Problem

The main issue with the questions in (2) is why it is such a difficult problem of construction Plato brings about in the passage. One can ask, as Gregory Vlastos (1988, 380) did, why is "the geometric example ostentatiously technical"? Or, why does Plato choose such a difficult problem just to illustrate the method of hypothesis? Could he not have picked up a much easier example? As Lloyd says (2004, 178), "when plenty of simpler ones would have made his chief methodological point," why did Plato go for this problem rather than any of the others? The crucial question here is this: what is so peculiar to this example such that Plato preferred and employed it in the dialogue?

To answer this question, one has to consider the first problem of geometry introduced at 82b in connection with the whole dialogue and its relevance to the second problem. Without going into the subtleties of the issues concerning the dialogue itself and the first problem, let me briefly outline what I think Plato intended to do with the first problem.

In the *Meno*, the dialogue between Socrates and Meno concerning the nature of *arete* comes to an impasse; for Meno brings into the dialogue at 80d what Socrates regards an "eristic argument," or what has been famously called the *Meno paradox:*

> Why, on what lines will you look, Socrates, for a thing of whose nature you know nothing at all? Pray, what sort of thing, amongst those that you know not, will you treat us to as the object of your search? Or even supposing, at the best, that you hit upon it, how will you know it is the thing you did not know? (Lamb 1967, 299)

Socrates at 80e, following Meno, reconstructs Meno's argument by saying that

> ...a man cannot inquire either about what he knows or about what he does not know? For he cannot inquire about what he knows, because he knows it, and in that case is in no need of inquiry; nor again can he inquire about what he does not know, since he does not know about what he is to inquire. [2] (Lamb 1967, 301)

Socrates then in order to clear the way for the discussion to proceed mentions a mythical story at 81c-d, which introduces two things: (i) The soul is immortal and transmigrates, and (ii) What ordinary people call learning is actually *remembering* (*anamnesis*); thus, knowledge is just remembering.

Socrates, as a response to Meno's wish that "if you can somehow prove to me that it is as you say, pray do so," brings into the dialogue the first problem of geometry. This problem is again one of construction. The question is how to double areawise a given square. Socrates initiates into solving this problem to a slave boy, who has no prior knowledge in geometry, other than some information concerning what a square is, and how to count. Socrates just directs certain questions in a particular order to the slave boy in order to bring forward the true opinion rested in the boy as the desired solution to the given problem of geometry.[3]

As it is clearly seen in the structure of Socrates' questioning the boy, the fact, which is instrumental for us, is that the way in which one remembers knowledge supposedly stored in one's soul can only work when right questions are asked in the proper order. If the teacher, Socrates that is, had not known the problem and the right solution to it, and also had had no idea which questions and in what order he had to ask the boy to see the desired solution, the boy could not have given birth to the correct solution. Asking correct questions in the proper order seems to be very important for enabling the boy to see the proper solution to the problem.

Moreover, another important point to see is that if Meno had not known the correct solution to the problem, he could have not judged if the boy had seen the right solution. So, unless one knows how to arrive at the solution by asking the proper questions, and knows that this is the correct solution to the problem, the search for something we do not know does not seem to have any teeth in practice.

So here Socrates seems to be demonstrating the very possibility of obtaining a new piece of knowledge by searching for it in a very particular case: the one searching for something who does not know gains this new piece of knowledge by himself/herself with the help of the questions that are asked by a particular person who knows what it is being searched for. By doing so, Socrates also resolves the second and the third part of the paradox: (i) Socrates has answered the question of "what sort of thing, amongst those that you know not, you will treat us to as the object of your search" by establishing that it is the last one which the boy arrived at by answering the proper questions asked in the right order; (ii) Socrates has also answered the question: even when "you hit upon it (the correct solution), how will you know it is the thing you did not know" by demonstrating that Socrates and Meno knew that it was the correct solution. Not only that, it is also by establishing that the solution proposed as the correct solution satisfies certain conditions for being the correct solution, though this is not discussed in detail in the dialogue.

It appears then as the next reasonable step for Plato to provide a cutting-edge for his claim, which even if we do not know a certain thing, it is still possible to search for it, is to see how his claim would behave when it tackles a problem, neither the solution nor the proper formulation of which none of the discussants knows. In other words, Plato wanted to illustrate how one obtains knowledge in the case of not knowing the problem proper as well as its correct solution. For this purpose, Plato had to choose a difficult problem. This means that the problem he had to choose is either its solution not being in hand, or being disputed about, since there might be many candidates for a solution and there might be no mechanical way of deciding which of them the desired one was. Better, to make his point more powerfully, the problem should have been the one that was not precisely or properly formulated yet at the time, i.e., it was not known if that formulation was the one to admit the correct solution when certain conditions are satisfied. This would be better for Plato's purpose because it could exhibit the power of the method of hypothesis more strongly.

It seems that Plato must have chosen the second problem of geometry not just to illustrate the method of hypothesis, but at the same time to complement the first problem of geometry, and beefed up his discussion about his claim concerning the very possibility of searching for something we do not know what it is that we are looking for. That is, he had to employ a problem, the solution of which necessitates the introduction of the method of hypothesis as to discover the desired solution to the problem. This line of thought makes more plausible the claim that Plato chose such a difficult problem as the second one for the reason that it would enable him to make his philosophical points in a better way, rather than the claim, as Vlastos (1988, 380) suggests, that Plato picked it up just to preen himself on his expertise in geometry. Thus, the second geometrical problem then becomes highly relevant to the first one and this, in turn, may enhance the understanding of the whole dialogue.

Sloppy and Casual Manner of Introducing the Geometrical Problem

Now, let us consider the questions in (3) which are about the way in which the problem was introduced. The problem was introduced in a sloppy and casual manner as if no further elaboration was needed. The following question then naturally arises: why is the problem then introduced in such a manner? From the way it was brought into the discussion, one could sense that each of the discussants knew which problem it was pointed to.

As an answer to the question above, one can again blame Plato himself for the ill formulation and presentation of the problem. But I do not think that this is a proper step in the right direction; for he was very careful with the first problem: Plato introduced it very neatly and stated clearly what it was sought as well as the conditions any solution for the problem to satisfy—although, he stated the proper formulation of the problem only towards the end of the session Socrates has with the slave boy, which is at 82b-85c.

It is most likely, as pointed above, that Plato was just following the geometrical tradition; namely, the problem should be one that was a very well known problem in the geometrical practice in Athens at the time. And the same consideration goes for the second problem as well; that is, since Plato introduced the first problem in the way that had been formulated and dealt with at the time, it is most likely that he brought the second problem about in the dialogue in exactly the same form and way that the problem has and was dealt with at the time. So, whatever formulation the problem has at the time, he must have introduced it in that form.

All of these considerations invite us to take into account the history of geometrical practice at the time along with the dialogue itself in order to reconstruct a conjecture concerning the nature of the second geometrical problem. The reason why the history of geometrical problems is needed here is the expectation that we may come across either the same or a similar geometrical problem in the geometrical practice at that time in Athens to the one introduced in the *Meno*.

PROBLEMS OF CONSTRUCTION

When one considers history, one cannot help noticing, as James Gow (1968, 161) does, that geometrical problems studied in Athens were those of a circle. The chief problems were the famous three problems of construction: (1') Quadrature of the circle: this is a problem of how to construct a square being equal in area to that of a given circle; (2') Duplication of the cube: this is also called the Delian problem (see Heath 1921, 244-46; Knorr 1986, 17-24; van der Waerden 1961, 159-62). It is a problem of how to construct a cube whose volume is twice that of a given one; and (3') Trisection of the angle: This is how to divide a given angle into three equal parts.

The famous three were thought of being "plane" problems.[4] This means that the only tools which were allowed to be used in solving these problems were the "plane" methods, that is, the classic (unmarked) straightedge and the compass.[5] Many mathematicians at the time attempted to solve these problems under these restrictions. According to Heath (1921, 218-19), they soon realised that "the three problems in question were not *plane*, but required for their solution either higher curves than circles or constructions more mechanical in character than the mere use of the ruler and the compass in the sense of Euclid's Postulates 1-3." However, while they were trying to solve these problems, they discovered a series of constructions by employing more powerful tools and techniques involving the so-called "solid" and "mechanical" methods as well as a method called "verging."

So these classical problems seemed to be immensely influential and instrumental in the development of geometry. This is pointed out by Gow (1968, 161) by maintaining that "it was mainly through a thousand attempts to solve these problems that new propositions[6] and new processes were discovered and geometry made daily progress." Heath (1921, 218) also underlies the same idea that

> ...the three problems in particular, the squaring of the circle, the doubling of the cube, and the trisecting of any given angle, were rallying points for

> mathematicians during the previous three centuries, at least, and the whole course of Greek geometry was profoundly influenced by the character of the specialised investigations which had their origin in the attempts to solve these problems.

When one considers both the so-called Delian problem and the first geometrical problem in the *Meno* at 82b, one could not help to notice a certain similarity between the two problems. It seems that the Delian problem has something to do with the incommensurable magnitudes. As reported by Gow (1968, 162), Carl Bretschneider makes the same observation: "…the duplication problem is due merely to this: the Pythagoreans had found that the diagonal of a square is the side of the square twice as large as that of which it is the diagonal, and they wished to find a similar law for the cube. It was well known in Athens before Plato's time."

This suggests that there is a certain parallelism between the duplication problem and Plato's first geometric example at 82b; for the example studied in the dialogue is the two-dimensional case of the duplication problem of the cube. This seems most likely to be the case. As stated above, Plato is then certainly just reporting the geometrical practice of his time in the case of his first example.[7]

The Quadrature of the Circle

Now let us see if the similar kind of parallelism can be found between the second geometrical problem in the *Meno* and any geometric problem studied in Athens then. My conjecture is that the best possible candidate for this problem is the quadrature of the circle.

There is a historical incident pointing in the right direction. This comes from a passage in one of Aristophanes' writings. The passage in question is in his comedy, *The birds*, which was staged in 414 BC (for the popularity of the problem, see, e.g., Heath 1921, 220-21 and Knorr 1986, 26). "This passage of Aristophanes," writes Heath (1921, 220), "is quoted as evidence of the popularity of the problem at the time." There, Aristophanes introduces into the play Meton, the astronomer and the discoverer of the Metonic cycle, who, under the restriction of the use of ruler and compasses, makes certain constructions "in order that your circle may become square." "If I lay out this curved ruler from above and insert a compass—do you see?—… by laying out I shall measure with a straight ruler, so that the circle becomes square for you" (for a different rendering, see Knorr 1986, 26, 44, and n. 46).

Setting aside certain problems with the passage, I think that Heath's (1921, 220-21) account seems to be a plausible one; namely, "this is a play upon words" and that "the word conveys a laughing allusion to the problem of squaring." It seems that those attempts in squaring the circle are either a laughing matter, or that Aristophanes represents them in that manner, probably due to a common belief that it is impossible to solve the problem. The problem must have been then taken a very *difficult* problem of construction. Secondly, it is most likely that people then should have had some information about those studies so that such an attempt could find a place in a comedy.

I think that a brief history of the problem until the time of Plato is needed here. Since we know that the *Meno* was written about 385 BC (Bluck 1961, 120)[8] and that the dialogue should have taken place between 402 and 401 BC (see Xenophon 2001, I.26-II.6.29 and Bluck 1961, 120) we should consider geometrical studies concerning this problem before 385 BC. There were several attempts to resolve this problem. These solutions before Plato's time were the ones presented by Antiphon the Sophist (about 480-411 BC), Hippocrates of Chios (about 470-410 BC), Hippias of Elis (460-400 BC), and Bryson of Heraclea (450-? BC). Since technical analyses of these solutions are already provided (for example, see Heath 1921, 220-35, 183-200; Knorr 1968, 25-39; and van der Waerden 1961, 131-36), I shall move on to consider the question of which one of these solutions of the circle quadrature is the one bearing any similarity upon Plato's second problem of construction in geometry in the *Meno*.

Due to the remarks by Aristotle (1984a and 1984b), which are at I, 185 a12-18 in *Physics* and at 11, 171 b10-20 in *Sophistical refutations*, my conjecture is that the most obvious and plausible candidate for Plato's second example in the dialogue seems to be the one that has something to do with Hippocrates' solution. In these passages Aristotle refers to and compares the errors in the attempts to square the circle by Hippocrates, Antiphon, and Bryson. In the passage of Aristotle's *Physics*, Aristotle says that Hippocrates proposed a false proof for the quadrature of the circle "by means of segments" or "by means of lunules." When Aristotle compares Hippocrates' attempt with that of Antiphon in the same passage, he says that "it is the duty of the geometer to refute the squaring of the circle by means of segments, but it is not his duty to refute Antiphon's proof." This is because the latter attempt does not stem from the first principles of geometry. The same reason can also account for the fact that Proclus [1970, 54 (66) and 167 (213)] mentions Hippocrates with certain lunules as a means of identifying him.

Another substantiating reason for my conjecture concerning the second problem of geometry in the *Meno* comes from the fact that Plato's example involves either *reduction* or of finding the *proper definition/determination/diorismos*,[9] which neither of the other solutions refers to.

Proclus [1970, 167 (213)], in his *Commentary,* ascribes the invention of *geometrical reduction* to Hippocrates, which involves

> … a transition from one problem or a theorem to another which, if known or constructed, will make the original [constative] evident. For example, to solve the problem of doubling the cube geometers shifted their inquiry to another on which this depends, namely, the finding of two mean proportional; and thenceforth they devoted their efforts to discovering how to find two means in continuous proportion between two given straight lines. They say that the first to effect reduction of difficult constructions was Hippocrates of Chios....

Moreover, Proclus [1970, 55 (66-67)], in his *Commentary,* also says that a particular Leon, the pupil of Neoclides, "discovered *diorismi* whose purpose is to determine when a problem under investigation is capable of solution and when it is not." Proclus [1970, 54-55(66)], while providing the chronology of the geometers, also says concerning this particular Leon that

> At this time [Plato's time] also lived Leodamas of Thasos, Archytas of Tarentum, and Theaetetus of Athens, by whom the theorems were increased

> in number and brought into a more scientific arrangement. Younger than Leodamas were Neoclides and his pupil Leon, who added many discoveries to those of their predecessors, so that Leon was able to compile a book of elements more carefully designed to take account of the number of the [constatives] that had been proved and of their utility.

Proclus basically says that Leon was later than Hippocrates and Plato. Heath (1921, 319) says that "of Neoclides and Leon we know nothing more than what is here stated." Heath goes on to say that "but the definite recognition of the *diorismos*, that is, of the necessity of finding, as a preliminary to the solution of a problem, the conditions for the possibility of a solution, represents an advance in the philosophy and technology of mathematics." On the other hand, Carl A. Huffman (2007, 5), in his *Archytas of Tarentum: Pythagorean, philosopher and mathematician king,* assigns ca. 400 as the birthdate for Leon.

Making a Hypothesis

There has been a discussion whether Plato was looking for *diorismos* or making a *geometrical reduction* in the said example. For example, Heath (1921, 303) takes the passage in question of the *Meno* as expressing a real *diorismos*, determination of not an actual solution of the problem but the possibility of its solution. He indicates this by saying that "the criterion sought by Socrates is evidently intended to be a real *diorismos* or the determination of the conditions or limits of the possibility of a solution of the problem whether in its original form or in the form to which it is reduced." Heath (1921, 303) then continues to say that

> ...the passage incidentally shows that the idea of a formal *[diorismos]* defining the limits of possibility of solution was familiar even before Plato's time, and therefore that Proclus must be in error when he says that Leon, the pupil of Neoclides, "invented *[diorismoi]* (determining) when the problem which is the subject of investigation is possible and when impossible," although Leon may have been the first to introduce the term or to recognize formally the essential part played by *[diorismos]* in geometry.

Besides, by referring to the fact stated at 87b1-2, Lloyd (2004, 172) writes that "we should be in a position to state whether or not the *inscription* is *im*possible." Lloyd (2004, 175) also maintains that "in the *Meno* inscription problem, the *diorismos* in the strict sense is clear." But he goes on to say that "however, in the *Meno* no distinction is made between *diorismos* and an actual solution, and indeed no clear reference is made to a *diorismos* at all."

With respect to the same issue, Knorr (1986, 71) writes the following:

> The *Meno* (86e-87b) introduces a method of reasoning "from hypothesis" which, but for its name, is identical to that of "reduction" as used, for instance, by Hippocrates in his attack on the cube duplication. The issue in the dialogue is to establish whether virtue is teachable. In the absence of a satisfactory

definition of what virtue is, Socrates proposes that one considers a hypothesis from which to pursue the examination of the main problem.

Knorr in the passage quoted here maintains that the method from hypothesis is identical to that of reduction. Moreover, he points out a certain parallelism between what Plato does in the dialogue and what Hippocrates offers with respect to the problem of the cube duplication.

Possibility of a Solution to a Geometrical Problem

However, Knorr in the next page explains why some "view the passage as discussing a 'diorism'." But more importantly he (1986, 73) directs our attention to a question which is to help us understand what is going on in the passage:

> A remarkable feature of the *Meno* passage is that it expresses the mathematical project as not the actual *solution* of a problem, but rather as the determination of the *possibility* of its solution. This has led many to view the passage as discussing a "diorism." But in the mathematical literature diorisms have the form of explicit conditions on the givens of the problem. … Although it is often the case that the analysis of a problem reveals the appropriate form of its diorism, nevertheless, the articulation of the diorism is quite different from the analysis or reduction of the corresponding problem. We thus have to explain why Plato here frames this example of problem reduction as if it were equivalent to the determination of possibility.

So the question Knorr thinks that one has to answer is "why Plato here frames this example of problem reduction as if it were equivalent to the determination of possibility." In order to facilitate an answer to this question, we need to reconsider the question why this problem of inscription is introduced in the dialogue. The main question in that part of the dialogue is whether or not *arete* is teachable; for Socrates seems to have given up searching for an answer to his basic question, that is, what *arete* is, in favour of Meno's main question, that is, whether *arete* is teachable. However, this is what appears to be the case. Socrates actually has not given up searching for an answer to his fundamental question; otherwise, the fact that he, up until that point of the dialogue, has been stubbornly trying to steer Meno in the direction of searching what *arete* is before they try to answer Meno's question could not make sense. So, Socrates seems to be devising a new strategy that is a new method, the method "from hypothesis" in order to keep up Meno's interest in the dialogue by seemingly giving in to Meno's wish. By doing so, on the one hand, Socrates would appear to be searching for an answer to Meno's question; on the other hand, he shall be still pursuing his initial aim. However, Socrates, being equipped with this new method, will be in a better position to look for an answer to the question of what *arete* is. It is because until that point, tackling with the question of what *arete* is, had not enabled them to find out what *arete* was. So the question as it stands is ill-formed and does not have a soluble form, yet. Thus, what Socrates does next is to try to reduce the question to such a form that will make it possible for them to formulate, not the actual answer but the question itself admitting of studying, which in turn may pave the way towards finding a desired result. In other words, by proposing a hypothesis, Plato aims at reducing the original inscription problem to the one so that a real *diorismos* of the problem can be searched for, and then a desired solution can be facilitated. So I think that what Plato here seems to be referring to is the problem reduction technique, but with a certain specification. But what is this certain specification?

We do seem to have some historical evidence in favour of this interpretation of the passage. Although we do not know for certain the birthdates of Hippocrates, Plato, and Leon, we know their estimated birthdates: for Hippocrates of Chios, it is estimated about 470 BC; for Plato, it is about 428/7 BC; and for Leon, about 400 BC, which is assigned by Huffman (2007, 5 fn1), seems to be the correct one. Since we know that the *Meno* was written about 385 BC, and that the dialogue itself should have taken place between 402 and 401 BC, and if the information given by Proclus concerning Leon as the one who discovered *diorismi* is correct, then it seems that when Plato wrote the dialogue, Leon could not have "discovered *diorismi*," and Plato could not have employed it in the *Meno*; for Leon could be only about fifteen years old.

However, there is another possibility that Heath might be right in claiming that Proclus must be in error in claiming that Leon *invented diorismos;*[10] for he thinks that the idea of *diorismos* should have been around even before Plato's time. But he (1921, 303) maintains that "Leon may have been the first to introduce the term or to recognize formally the essential part played by *diorismoi* in geometry." If this is the case, then there are two cases possible for the *diorismos* discussion. Firstly, it could be that although the idea of *diorismos* might have been recognised, there might have been some confusion concerning its applicability as a formal tool in geometrical activity. Secondly, it could be that the essential role *diorismos* to play formally in geometry might have been realised as a proper definition of a problem determining whether it admits a solution or not, but this could only be achieved by means of reductions (viz., when reduction and *diorismos* complement each other) that they could function their proper role in geometry. That is the specification, I think, why Plato beefed up the reduction technique.

I believe that these two possible cases can enable us to provide a satisfactory answer to the question Knorr asks: "why Plato here frames this example of problem reduction as if it were equivalent to the determination of possibility?" However, I think that we can choose one of these possible cases as the most likely answer by bringing into the discussion the following considerations: that there is a parallelism between what Socrates did when he was asking questions to the slave boy to facilitate the desired solution to the duplication problem at 82b and what he means at 86e by the method of hypothesis; namely, Socrates seems to have already spelled out implicitly how the method of hypothesis works in solving the first problem of geometry.

To elucidate my point here let me briefly outline what Socrates is doing in the case of the first problem. Socrates not only knows the correct solution to the problem, but also knows how to reach at the solution in question. Since Socrates would like the boy to find the solution himself, Socrates navigates, by asking the right questions at the right times, for the boy to form a series of hypotheses, which are to be formed at certain stages of the search for the correct solution, to make the boy realise how to see the correct solution.

Socrates initially at 82d-e directs certain questions to the boy in such a way that the boy would see a certain relationship between the length of the side of a square and its area: "Come now, try and tell me how long will each side of that figure be. This one is two feet long: what will be the side of the other, which is double in size?" That is, the longer the length, the bigger the area of the square or the shorter the length the smaller its area. So the very function of the questions at this stage of the investigation that Socrates asks is to make the boy see this relationship. Socrates expects the slave boy, by having realised this relationship, to form a certain hypothesis by which the boy would think to find the solution. This is the first hypothes that the boy would form to solve the duplication problem, one has to double the length of the side of a square, i.e., the area of a square is directly proportional to the length of its side. Socrates is well aware of the fact that the hypothesis could not lead the boy immediately to the solution. Nevertheless, it is instrumental in the eyes of Socrates for the boy to form later on a better or more productive hypothesis. In other words, Socrates thinks that this is a certain stage that the boy has to go through.

The boy is certain that he got the solution by means of this hypothesis. However, Socrates by employing *reductio ad absurdum* manages to abolish the solution put forward by the boy: it is not twice, but actually four times, bigger than the area of the original square; so it cannot be the solution. The boy now understands that the hypothesis is wrong, but that the relationship between the area of a square and the length of its side by the help of which he had constructed the hypothesis, seems to be still standing; for he appears to be on the right track, but not quite on the right point. Moreover, Socrates' questions at 83c-e direct him in this direction, too:

> Socrates: What line will give us a space of eight feet? This one gives us a fourfold space, does it not?
>
> Boy: It does.
>
> Socrates: And a space of four feet is made from this line of half the length?
>
> Boy: Yes.
>
> Socrates: Very well; and is not a space of eight feet double the size of this one, and half the size of this other?
>
> Boy: Yes.
>
> Socrates: Will it not be made from a line longer than the one of these, and shorter than the other?
>
> Boy: I think so.
>
> Socrates: Excellent: always answer just what you think. Now tell me, did we not draw this line two feet, and that four?
>
> Boy: Yes.
>
> Socrates: Then the line on the side of the eight-foot figure should be more than this of two feet, and less than the other of four?
>
> Boy: It should.
>
> Socrates: Try and tell me how much you would say it is?

So the boy can still rely on the relationship between the area and the length of a square to form another hypothesis or better to correct the first hypothesis; for the boy now knows that the side of the square they are looking for is shorter than four bigger than two. But this time he has to work out a hypothesis that will give the correct ratio between the area and the length. However, there does not appear any alternative other than to have the length to be three, since their theory of proportion is a numerical one. This alternative does not work either, for Socrates establishes that the area of this square is nine, which is not the one they are looking for. So the boy understands that the second hypothesis has to go as well. Socrates is now well aware of the fact that the boy feels himself in a cul-de-sac, so to speak.

Next, Socrates, being aware of this fact, changes the nature of the questions he has been asking the boy. Actually, this change starts with asking questions at the second try at 83c by saying that "what line…," but this *line* term is always associated with length. By this time Socrates drops the length part altogether at 83e-84a: "But from what line shall we get it? Try and tell us exactly; and if you would rather not reckon it out, just show what line it is."

From the beginning Socrates's questions are algebraic in nature, but from the passage at 83e-84a on, they turn out to be geometrical in character. In other words, these questions have made the boy realise that he has to change the very nature of the hypotheses he has so far formed: he has to come up with a hypothesis in terms of geometry rather than those of algebra. But the boy does not feel that he can do so this time: "Well, on my word, Socrates, I for one do not know." Only after some of Socrates' geometrical constructions that the boy discovers the answer to the question: "From what line shall we get it?" The boy's answer is: "From this" (85a). By finding the correct answer to this question, the boy now is able to provide with the proper definition, *diorismos*, of the doubling problem: "according to you, Meno's boy, the double space is the square of the diagonal" (85b).

Let me iterate more acutely what is going on in this passage between Socrates and the boy: the dialogue between Socrates and the slave boy can be taken as an example of teaching someone something new. What in this passage Plato seems to be arguing for is that there are certain stages that a learner—the slave boy in this case—has to go through finding the correct solution to the problem:

(a') Since Socrates knows the correct solution to the problem, he can work out which questions should be directed to the boy in a certain order so that the boy could see a certain seemingly invariant relationship between certain elements of the problem. The boy, having realised this relationship, could form a hypothesis by which the boy would reduce the initial problem to another, a solution which could easily be offered on this hypothesis. However, the fact that this solution could not be the correct solution can be established by means of *reductio ad absurdum*. By this disproof, the boy would be expected to realise that the relationship must have been a good one, for the first hypothesis constructed on the basis of this relationship seems to have put the boy on the right track. It is because the boy knows that the side of the square they are looking for is shorter than four bigger than two. So the boy would have every reason to go on searching for the correct solution.

(b') At this stage, what the boy must do now is to form another hypothesis on the basis of the same relationship by means of which the boy could work out the correct ratio between the length of the side of, and the area of the square. However, since the boy employed the very naive theory of proportion, that is, the numerical one, and there was only one option available to him, he went for that one, hoping that it could provide the correct solution. That solution was established as being not the correct solution as well with the help of *reductio ad absurdum*.

(c') The failure of the second hypothesis must have been a great deal of shock for the boy; for he would probably have had the feeling of having come to a dead end. But this is good for the boy; for he would now be expected to realise that with the failure of the second hypothesis, the naive theory of proportion could not deliver the correct ratio between the length of the side and the area. So, I am not sure if this should be taken as the criticism Plato levels on Hippocrates' reduction of the doubling problem, but the boy should be aware of the fact that he could not rely on this naive theory of proportion for bringing about the correct ratio. The new hypothesis he would form, which he was presently not able to do so, should be able to take care of all these difficulties. However, this requires a good deal of knowledge of geometry, which he did not have. Thus, the boy has to make use of an expert's knowledge, and that help came from Socrates. This is the stage Socrates changed the nature of his questions. With the help of Socrates' geometrical constructions, the boy saw the correct solution, but with the correct solution the boy discovered the proper *diorismos* of the problem. So it seems for Plato that the method of hypothesis is a procedure combining of reduction by means of hypotheses and finally finding the real *diorismos* of the problem.

Coming back to my conjecture concerning the nature of the second problem of geometry Plato refers to at 86e in the *Meno*, in the light of all these considerations given above in conjunction with Aristotle's remarks, it seems very plausible that Plato, at 86e in the *Meno*, most likely refers to a similar strategy in the case of the problem of the quadrature of the circle by Hippocrates; namely, to cast the problem "into an alternative form permitting the application of a wider range of techniques then known" (Knorr 1986, 29).

Moreover, when Aristotle (1984c) refers to the same procedure as a syllogistic proof in his *Prior analytics*, he writes in II Book at 25, 69 a20-36:

> By reduction we mean an argument in which the first term clearly belongs to the middle, but the relation of the middle to the last term is uncertain though equally or more convincing than the conclusion; or again an argument in which the terms intermediate between the last term and the middle are few. For example, let *A* stand for what can be taught, *B* for knowledge, *C* for justice. Now it is clear that knowledge can be taught; but it is uncertain whether virtue is knowledge. If now *BC* is equally or more convincing than *CA*, we have a reduction; for we are nearer to knowledge, since we have made an extra assumption, being before without knowledge that *A* belongs to *C*. Or again suppose that the terms intermediate between *B* and *C* are few; for thus too we are nearer knowledge. For example let *D* stand for squaring, *E* for rectilinear figure, *F* for circle. If there were only one term intermediate between *E* and *F* (viz. that the circle is made equal to a rectilinear figure by the help of lunules), we should be near to knowledge. But when *BC* is not more convincing than *AC*, and the intermediate terms are not few, I do not call this reduction; nor again when *BC* is immediate—for such statement is knowledge.

When writing the passage above it is most likely that Aristotle had the *Meno* in mind; for the topic and the intriguing example Aristotle speaks of are clearly the ones discussed in the *Meno*. Moreover, Gow (1968, 170, n. 1) refers to the same passage by saying that "D is capable of quadrature; E is a rectilinear figure; Z a circle. All E is D, but that Z is E is one step short of certainty, since we know only that a circle with a lune is equal to a rectilinear figure."

Study on Lunules

About Hippocrates' solution we basically rely on Simplicus' testimony and thus two accounts he provides in his commentary on Aristotle's *Physics*, the one by Alexander of Aphrodisias and the other one by Eudomus (see Heath 1921, 183-201 and Knorr 1986, 25-39). Although there are some subtle issues with these accounts, since the translations and technical analyses and details are readily available, I shall just provide a brief summary of Hippocrates' solution. The simpler account of Hippocrates' study on lunules, that is, Alexander's, will suffice to make my point here.

Two cases of lunules are provided. However, the first case is the one we should be concerned with for the purpose of the present discussion. In this case, one starts from constructing a right isosceles triangle. Then on each of its sides a semi-circle is drawn. One knows that the square on the hypotenuse is equal to the sum of the squares on the two legs. One also knows that circles are as the squares on their diameters. One can then infer that the semi-circle on the hypotenuse will be equal to the semicircles on the legs. Now if one removes the area, which is common to the large semicircle on the hypotenuse and the two smaller ones on the legs, one will find that the isosceles triangle is equal to the two lunules drawn over the legs. Then "by equating the lunules to a rectilinear figure we have effected their quadrature" (Knorr 1986, 30). So, the case looks like the one in Figure 4:

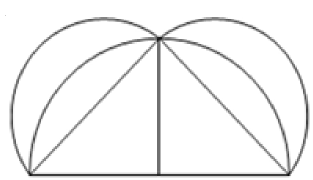

Fig. 4

ATTEMPT AT A SOLUTION TO THE PROBLEM

Now I would like to outline my attempt to reconstruct the second geometric example of *the Meno*. We start from a given area/figure, say X. The task is then to inscribe this area/figure in the form of a triangle in the given circle AEBF (see Figure 5).

One can draw two diameters AB and EF cutting each other at a right angle at O. Then the triangle to be inscribed into the given circle is AEF. The rectilinear figure area OBCE, which is formed by radii, OB and OE, and BC and EC, is the one to be applied on the radius of the circle. The rectilinear figure by which it falls short is also a rectilinear

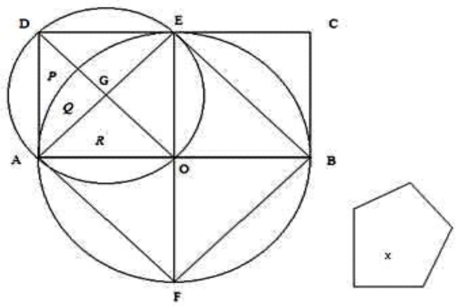

Fig. 5

figure—in this case it is a square, that is, AOED—and equal to the rectilinear figure, OBCE, we initially started with. We can form an isosceles triangle AEB by drawing two lines AE and EB. We can then draw another circle through ADEO by forming the line AE as its radius and the point G, as its centre. We also have another triangle AOE, a right-angle triangle. We have thus formed three areas and let us denote the area enclosed by the sickle ADE by P; the area of the right angle triangle AOE by R, and the area remaining between these two areas P and R by Q in Figure 5.

We are now in a position to compare the areas in Fig. 5. From Hippocrates' study we know that the bigger semicircle AEB on the hypotenuse AB of the square AFBE is equal to the smaller circle ADEO with the centre G on the leg AE. Then the half of the semicircle AEB is equal to P + Q. We can then say that P + Q equals to Q + R. Now if we remove the area, that is, Q, which is common to the large semicircle AEB and the smaller one ADE, we get that the isosceles triangle AOE equals to the lune arching over the ADE which is denoted by P in the Figure 5. Namely, Q=R. What we have done until here is no different from Hippocrates' reduction of the quadrature problem.

We can answer the question if it is possible to inscribe a given area in the form of a triangle in a given circle by forming the hypothesis that when this given area X be applied on to the diameter of the given circle AEBF in Figure 5, if the area falls short by an area such as the one stretched itself, we can inscribe it as a triangle in the given circle. Not only this, we can also know how to construct it, though we are not at the moment interested in how to construct it in the given circle. However, if the remaining area is smaller than the one applied, we can say that the area can be inscribed, though if it is not numerically proportional to the triangle we have thus constructed, we cannot know for sure how to construct it.

So, the following question naturally arises: are these conditions specified by the hypothesis powerful enough to enable us to answer the question in the affirmative sense if these conditions be met, if not, in the negative sense? Namely, can it always be the case that it cannot be inscribed if the area X is larger than the remaining area? To answer this question, we have to form another hypothesis in the form of a question: is there any larger area inscribable in the given circle as a triangle?

For this, since we know how to construct an equilateral triangle by Euclid's *Elements* Bk. I, Prop. 1, let us construct it as ACB in Figure 6 by taking A and B centre

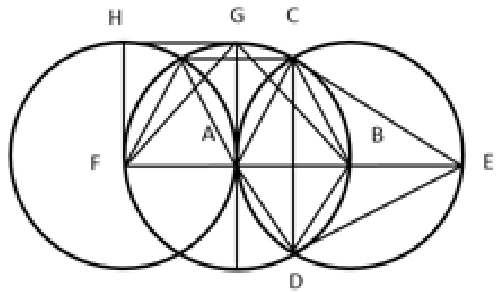

Fig. 6

points, respectively, and AB as their radius. Let us define another triangle CED by connecting first C and E, then D and E.

What we should be doing now is to establish whether or not CDE is larger than 2FAG. However, this will be a bit tricky, for we should be working with the numerical theory of proportion. So the demonstration will be sketchy. It would not be difficult for a Greek geometer of the time to establish that this triangle CDE is an equilateral triangle as well. So, all the triangles CBD, CBE, and DBE are equal to each other. Moreover, it can be established that ACB=CBD and thus, 3ACB=CDE. And then we can infer that 3ABC is larger than 2FGA since there are 3 ACB equilateral triangles to be constructed in the semi-circle FGB in Figure 6 and these cover more area than 2FGA or FGB. Thus, the triangle CDE is larger than 2FGA. Or we can establish the same conclusion by demonstrating that both CE and DE are longer than AG and the height of the triangle CDE is longer than the height GA of the triangle FGA.

Knorr (1986, 92-94) in his 1986 book provides more detailed account of the demonstration that the bigger equilateral triangle CDE is bigger areawise than the triangle, FGB, on the base of the radius. By establishing that the triangle CDE is bigger than FGB, one can maintain that the initial hypothesis is wrong; for there is a triangle bigger than the triangle constructed on the basis of the first hypothesis. So it appears that we seem to be still on the right track by means of our initial hypothesis, but we have not managed to find the desired solution, yet. So we need to form another hypothesis by which we can go further than we have with the help of the first hypothesis. By this new hypothesis we can reduce the initial problem to the one more manageable.

The second hypothesis should be: what is the largest triangle one can inscribe in a given circle? Or is the triangle CDE is the largest one? By this hypothesis, if we find the correct answer to this question, we can determine the upper limits of the correct solution. Knorr (1986, 92) provides the answer to this question by saying that we know that "the equilateral triangle is the greatest of triangles inscribed in the same circle." That is, the triangle CDE is the largest triangle inscribable in this given circle. Knorr also maintains that "it seems to me that no feature of this proof of the diorism would have posed any difficulty for a Greek geometer circle." So we now know that if any given area is larger than the area of this equilateral triangle, we can say that it cannot be inscribed in this circle in the form of a triangle.

However, we have a problem here, too; for a Greek geometer of the time was not in a position to be able to make a comparison between the areas of equilateral triangles and those of squares and those of circles; for they were still working with the numerical theory of proportion. I think that while they were working on this, Plato wrote the passage referring to these issues; namely, when Plato refers to a problem of geometry at 86e, he is not talking about just one problem, but about the whole procedure of solving a problem we have talked about above. And the problem that initiates the whole procedure is most likely the case of the circle quadrature by Hippocrates. Moreover, Plato's intention seems to be not only to speak of this problem, but the procedure associated with the problem, which he devised most likely on the basis of the Socratic method of *elenchus* and the techniques the Greek geometers were employing at the time.

CONCLUSION

This method, as I argued above, is implicitly cashed out in dealing with the duplication problem. So, it is not surprising to see the same strategy being employed in the case of the second problem of geometry appearing at 86e. The same strategy takes care of the first part of Meno's paradox as formulated by Socrates. Specifically, in the case of someone teaching somebody something new, it establishes that the search for new piece of knowledge is not only possible, but also at the same time the following two claims which are the second and the third part of Meno's paradox: the object of the search is the problem, the diorismos of which the slave boy comes up with at the end of the search, i.e., when the slave boy finds the desired solution, and the correct solution of the problem, the solution which is looked for, is the one that is obtained at the end of the search; for not only that Socrates and Meno knew that this was the correct solution, but also that the solution proposed as the correct solution satisfies certain conditions, given by the diorismos of the problem, for being the correct solution. Plato argues that the same method could take care of the case that none of the discussants knows neither the diorismos of the problem, the correct solution of which they are searching, nor its correct solution. Plato must have been aware of the fact that there is no mechanical guarantee that we could achieve all these aims, but Plato (81d) seems to be arguing that "if we have courage and faint not in the search" in the way that his method specifies, there is a hope that we may do so.

NOTES

1. I would like to express my gratitude to Professor Alexander Mourelatos for enlightening talks and comments on an earlier draft of this paper. I also wish to thank Tonguc Seferoglu for helpful discussions.

2. One might presume that Socrates here seems to be evading the second and the third parts of the paradox as formulated by Meno. However, as it will be clear below, this is not the case; for Socrates considers them separately in either case of his reformulation of the paradox, i.e., "a man cannot inquire either about what he knows or about what he does not know."

3. One might think that Socrates' intention is actually to prove the *anamnesis* theory, or the claim that the soul is immortal, by asking questions to enable the slave boy to come up with the desired solution to the given problem. However, one can argue that Socrates' desire cannot be this; for he says at 86b:

> Most of the points I have made in support of my argument are not such as I can confidently assert; but that the belief in the duty of inquiring after what we do not know will make us better and braver and less helpless than the notion that there is not even a possibility of discovering what we do not know, this is a point for which I am determined to do battle, so far as I am able, both in word and deed. (Lamb 1967, 323)

So, it seems that what Socrates really wants to do here is to resolve the aporia in order to carry on with the dialogue, and to establish the very possibility of searching for a new piece of knowledge. However, be as it may, what I would like to argue for is neutral to what Socrates is actually aiming at here.

4. According to Pappus (cf. Thomas1957, 348-49) in Book VI of *Collection*, the mathematical problems of the era were classified into three levels of complexity. They were grouped with respect to the types of curves employed in the construction in order to solve them. Corresponding to these three types of curves, there were three types of problems: plane, solid, and linear. *Plane problems* could be solved only by the employment of plane curves; namely straight lines and circles. *Solid problems* could be solved by means of one or more solid curves, that is, conic sections–ellipse, parabola, and hyperbola. Thus, one needed to use more than straight lines and circles in order to solve solid problems. Lastly, to solve the *linear problems* one appealed to the methods mentioned above as well as to more complicated curves than those of solids involving shapes like spirals, quadratices, cochloids, and cissoids.

5. Heath (1921, 175-76) writes that "he [Oenopides] may, for example, have been the first to lay down the restriction of the means permissible in construction, the ruler and compasses, which became the canon of Greek geometry for all plane constructions, i.e., for all problems involving the equivalent of the solution of algebraic equations of degree not higher than the second."

6. Rolando Gripaldo (2003, 2011) rejected the concept *proposition* and replaced it with the concept *constative*.

7. One can always bring in some historical data confirming this conjecture, historical evidence concerning the friendship Plato had with Archytas of Tarentum, who provided a solution for the Delian problem [for more, see Marcus Tulluis Cicero (1983, 5.29.87; 1977, 1.10.16; and 2005, 1.17.39)].

8. Moreover, Guthrie (1975, 236), for example, gives the date between 386-382. Thomas (1980, 22) accepts the date given by Bluck. But there is one exception: Morrison (1964, 42) thinks that it was written before Plato's first visit to the west.

9. The term *diorismos*, according to Heath (1956, 129-131), was employed in two technical senses in Greek geometrical activity. One of its uses is to mean the particular *enunciation* of a Euclidean [constative]. In this sense, it means *definition* or *specification*, that is, "a closer definition or description of the object aimed at, by means of concrete lines or figures set out in the *ekthesis* instead of the general terms used in the enunciation; and its purpose is to rivet the attention better…." The other technical sense of the word is "to signify the limitations to which the possible solutions of a problem are subject." Heath says that Proclus knows both senses of the word [see Proclus 1970, 159 (203), 162 (208), and 158 (202)]. However, Heath (1956, 130-31) maintains that Pappus uses the word

in the latter sense only. Moreover, one might see from Heath's discussion how these two senses of the use of the term might have come to be merged.

10. Proclus (1873, 66) uses the word *eurein* in the passage. Heath takes it to mean as "invented," but Morrow (Proclus 1970, 55) translates it to mean as "discovered." However, it means in its primary sense "to find"; Heath, probably because of this, regards Proclus being in error.

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