Φιλοσοφια
Volume 17, 2:2016

GENERAL CONDITIONALS IN STOIC LOGIC[1]

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It is often thought that Stoic logic is only a particular constatival logic. In this paper, I will try to show that this idea is not correct and that the Stoics addressed at least general conditionals. To do this, I will take into account arguments such as those raised by Josiah B. Gould, W. H. Hay, and Robert R. O'Toole and Raymond E. Jennings. Likewise, I will propose that, if the concept of "Feeder schemata"—conjunction introduction and elimination rules—of the mental logic theory is considered, the points or aspects that are unclear in the ancient sources can be easily understood.

INTRODUCTION

Stoic logic is usually thought to be a *particular* constatival logic[2] linked to truth values (e.g., Bobzien 1996; Barnes, Bobzien, and Mignucci 2008), and mainly based on the five "indemonstrables" attributed to Chrysippus of Soli (i.e., *Modus Ponens*, *Modus Tollens*, *Modus Tollendo Ponens*, and the two versions of *Modus Ponendo Tollens*). Obviously, this is a very simplified view of Stoic logic, since it includes and refers to many more points or aspects that deserve analysis and consideration. However, the problem that will be primarily addressed in this paper is that related to general conditionals.

Indeed, arguments such as those of Josiah B. Gould (1970) allow us to suppose that the Stoics, or more specifically Chrysippus, considered *general* conditional constatives. My thesis is that Stoic logic could have treated such conditionals as sentences akin to those that are named "universally quantified constatives" in modern logic and admitted the possibility to infer particular sentences from universal sentences. But the thesis is not only based on Gould's (1970) arguments, but also on other evidences coming from works such as those of W. H. Hay (1969), Robert R. O'Toole and Raymond E. Jennings (2004), and the ancient sources. In this way, the aim of this paper is to analyze and explore those evidences in order to show that it can support my thesis. This point is important because our knowledge of the Stoics' logic is still fragmentary and, while there are important reconstructions of it (e.g., Bobzien 1996), some aspects of this ancient logical system remain unclear and need to be clarified.

The basic idea is that it is legitimate to suppose that Stoic logic enables us to generalize particular or singular conditional constatives and, by virtue of that generalization process, to come to a general conditional constative. The general conditional would consist of the conjunction of the particular conditionals, being each particular conditional a "conjunct." Likewise, the conjuncts could be separated from the general constative and, therefore, it would also be possible to infer particular conditionals from general conditionals.

As it can be noted, I am speaking about an induction-deduction process that can be attributed to the Stoics. The only difficulty of this idea is that a conjunction, which is very relevant in my account, is not addressed in detail by Stoic logic (at least that is what can be concluded from the ancient sources). Nevertheless, I will try to argue that this problem can be easily solved if the theses of a current psychological or reasoning theory are taken into account. That theory is the mental logic theory (e.g., Braine and O'Brien 1998a; O'Brien 2009, 2014; O'Brien and Li 2013; O'Brien and Manfrinati 2010) and, as it can be seen below, has clear and evident links and relations with Stoic logic.

This paper has two sections. The first exposes the characteristics that, in my view, general conditionals have in Stoic logic. Such characteristics refer to the fact that those conditionals could be considered to be really conjunctions of particular conditionals. The second one explains the actual role of conjunctions in this very logic and describes in more detail their relationship with conditionals.

THE STOICS AND GENERAL CONDITIONALS

Although it is true that "…Stoic logic is in its core a [particular constatival] logic" (Barnes, Bobzien, and Mignucci 2008, 92), there is no doubt that Stoic logic does not reject general conditionals. As indicated by O'Toole and Jennings (2004), we do not have many examples (which is what can lead one to think that Stoic logic is basically a particular constative), but there are some of them. In particular, they quote two of them, that of Cicero in *De fato* 12 and that of Sextus Empiricus in *Adversus mathematicos* 11.8. The exact conditional of the first one is "If anyone (*quis*) was born at the rising of the dogstar, he will not die at the sea" (O'Toole and Jennings 2004, 495), and that of the second one is "If something (τι) is a man, that thing (εκειϖο) is a rational, mortal animal" (O'Toole and Jennings 2004, 495).

Based on Gould (1970), O'Toole and Jennings (2004) seem to claim that it was possible in Stoics thought to come to general conditionals such as the two just mentioned by means of a process akin to that of induction. In the first case, this means that a general constative such as "If anyone was born at the rising of the dogstar, he will not die at the sea" can be drawn from the observation that many people that were born at the rising of the dogstar did not die at the sea. On the other hand, in the second case, the idea means that a general constative such as "If something is a man, that thing is a rational, mortal animal" can be concluded from the observation that many "things" that are men are also rational and mortal animals.

Thus, they remind us, according to Hay (1969, 151), that if we take into account the constative indicated by Sextus Empiricus in *Adversus mathematicos* 8.305, i.e., "If Dion is walking, Dion is moving" (O'Toole and Jennings 2004, 495) and the general conditional that could be linked to it, i.e., "If someone is walking, he (or that one) is moving" (O'Toole and Jennings, 2004, 495-96), we can think that, indeed, Stoic logic includes and considers universal quantification, since the constative in *Adversus Mathematicos* 8.305 seems to be a particular example of this latter general conditional.

As it is known, Charles H. Kahn (1969, 164) noted some drawbacks in constructing arguments in this way, since they can lead us to assume that either Stoic logic is only a "sterile" constatival logic or else it is a "defective" logic that does not provide an adequate explanation of quantification. O'Toole and Jennings (2004, 196) seem to respond to these objections, arguing that, firstly, a particular constatival logic does not need to be classical constatival logic and that, secondly, Stoic logic can explain how a particular conditional can be derived from a general conditional. To show that that latter idea is correct is a goal of this paper. However, for the moment, what is important is to analyze the process enabling us to come to general conditionals from particular or singular conditionals.

Maybe it is easier to account for that process if we assume some equivalences, for example:

A: Someone is walking
B: The individual indicated in A is moving
a_1: Dion is walking
b_1: Dion is moving

If we adopt "—>" to stand for a conditional relationship (i.e., "implies"), the problem is to explain how it is possible to come to A —> B from singular constatives such as a_1 —> b_1. Obviously, it would be necessary to consider *n* constatives with the forms a_1 —> b_1, a_2—> b_2, a_3—> b_3,… , a_n —> b_n, which would allow concluding the general constative, A —> B. Of course, those *n* constatives would refer to different particular individuals. For example, in addition to the meanings indicated for a_1 and b_1, "a_2" could mean that "Socrates is walking," "b_2" that "Socrates is moving," "a_3" that "Plato is walking," "b_3" that "Plato is moving," and so on. Therefore, the entire inference would have a form similar to this one:

a_1 —> b_1
a_2 —> b_2
a_3 —> b_3
…
a_n —> b_n
———————
Ergo A —> B

However, O'Toole and Jennings's (2004, 496-97) idea does not appear to be that simple. They seem to consider that the general conditional ("all") consists of the conjunction of the particular conditionals, which are transformed into the different conjuncts of a formula with *n* conjuncts. So, we would have to suppose a process such as the following:

a_1 —> b_1

$(a_1 \longrightarrow b_1) \cdot (a_2 \longrightarrow b_2)$
$(a_1 \longrightarrow b_1) \cdot (a_2 \longrightarrow b_2) \cdot (a_3 \longrightarrow b_3)$
. . .
$(a_1 \longrightarrow b_1) \cdot (a_2 \longrightarrow b_2) \cdot (a_3 \longrightarrow b_3) \cdot \ldots \cdot (a_n \longrightarrow b_n)$

Ergo $A \longrightarrow B$

Where "·" denotes a conjunction.

In addition, as a result, we would accept this definition:

$$A \longrightarrow B = [(a_1 \longrightarrow b_1) \cdot (a_2 \longrightarrow b_2) \cdot (a_3 \longrightarrow b_3) \cdot \ldots \cdot (a_n \longrightarrow b_n)]$$

This appears to be O'Toole and Jennings's (2004, 496-97) argument and, in my view, its only difficulty is that a conjunction does not play a relevant or important role in Stoic logic. Nevertheless, I will try to solve this problem in the next section.

CONJUNCTIONS IN STOIC LOGIC

Based on the previous section and O'Toole and Jennings' (2004), it appears to be obvious how a particular conditional such as $a_i \longrightarrow b_i$ could be deduced from $A \longrightarrow B$. Given that $A \longrightarrow B$ is equivalent to a formula such as $(a_1 \longrightarrow b_1) \cdot (a_2 \longrightarrow b_2) \cdot (a_3 \longrightarrow b_3) \cdot \ldots \cdot (a_i \longrightarrow b_i) \cdot \ldots \cdot (a_n \longrightarrow b_n)$, we only need a procedure enabling us to separate the different conjuncts of a conjunction. As it is well known, Stoic logic has no a clear rule or schema for eliminating conjunction. Nonetheless, I think that this difficulty can be overcome if the findings of the mental logic theory are taken into account.

But before going to such findings, it is, perhaps, also appropriate to describe in greater detail the actual role of conjunction in Stoic logic. It is true that a conjunction is not as important as a disjunction or the conditional in that logic. However, that does not mean that a conjunction has no importance for the Stoics. As it is also well known, that a conjunction is essential in one of Chrysippus' "indemonstrables," in particular, in one of the two versions of *Modus Ponendo Tollens*. That version can be formally expressed as follows:

$\neg (a \cdot b)$
a

Ergo $\neg b$

Where "¬" denotes denial.

This version of *Modus Ponendo Tollens* reveals that a conjunction does play a role in Stoic logic. Although, at least as far as I know, they never proposed an explicit rule, the Stoics allowed introducing conjunctions. The example mentioned above can be very useful to clarify this point. As indicated, Cicero says in *De fato* 12: "*Si quis (verbi* *causa) oriente Canicula natus est, is in mari non morietur*" ["if anyone was born at the rising of the dogstar, he will not die at the sea" (translation by O'Toole and Jennings 2004, 495)]. But then he refers to the discussion between Chrysippus and Diodorus Cronus and states that, if the latter constative is true, it is also true that "*Si Fabius oriente Canicula natus est, Fabius in mari non morietur,*" that is, that Fabius cannot die at the sea if he was born at the rising of the mentioned star. Thus, Cicero argues that the constatives "Fabius was born at the rising of the dogstar" and "Fabius will die at the sea" are inconsistent. Nonetheless, given that Cicero considers "Fabius was born at the rising of the dogstar" to involve "Fabius exists," he finally claims that "*Ergo haec quoque coniunctio est ex repugnantibus: 'Et est Fabius, et in mari Fabius morietur', quod, ut proposition est, ne fieri quidem potest*" ("So, 'Fabius exists and Fabius will die at the sea' is an incoherent conjunction, which indicates an impossible fact." Beyond the discussion involved in Cicero's text, what is important for this paper is that, by affirming that the constative (conjunction) "Fabius exists and Fabius will die at the sea" is inconsistent and impossible, he uses the word "*coniunctio,*" which suggests that, although they did not clearly explain that, the Stoics admitted the possibility to introduce conjunctions from simple constatives taken as conjuncts.

In this way, it seems that it is not hard to account for the process why several particular conditionals can become a general conditional. The problem is now then to explain the reverse process, i.e., the process why a particular conditional can be drawn from a general conditional. It is clear that the Stoics admitted the latter process, and that it can be noted not only by means of the example referring to Dion as commented above, but also by means of examples akin to that of Fabius in this section. From "If someone is walking, that one is moving" can be derived from constatives such as "If Dion is walking, Dion is moving," and from "If anyone was born at the rising of the dogstar, he will not die at the sea" can be deduced from the constatives such as "If Fabius was born at the rising of the dogstar, Fabius will not die at the sea." Based on the ancient sources, some more examples are possible, but I think that these two are illustrative enough to show that this kind of derivation is allowed in Stoic logic.

As far as this issue is concerned, O'Toole and Jennings (2004) seem to state that, if Stoic logic enables us to separate the conjuncts from a conjunction (remember that that logic has no explicit rule for doing so), then that logic enables us to derive a particular conditional from a universal conditional as well. Therefore, what is needed here is to describe the exact way in which that could be done. In my view, the mental logic theory can help.

"Mental logic" is a current theory on human reasoning. It claims that the human mind reasons by means of rules, but its idea is not that the human mind knows and applies all of the rules of standard constatival calculus or that follows the rules of the natural deduction calculi such as that of Gerhard Gentzen (1935) in a natural way. The mental logic theory is based on experimental results and, according to it, such results show that, although people use many schemata valid in classical logic, they do not often use all of the schemata valid in standard logic. In fact, the results indicate that it is necessary to distinguish between different types of schemata. As explained in Martin D. S. Braine and David O'Brien (1998b, 79-84), there are "Core schemata," "Feeder schemata," and "Incompatibility schemata." However, the types that are interesting for this paper are the Core schemata and the Feeder schemata. Individuals always use the Core schemata in a rapid and automatic way. This is interesting because there are only seven schemata in Braine and O'Brien (1998b, 80) and most of them seem to be versions of Chrysippus' "indemonstrables" (Lopez-Astorga 2015, 3-12). For example, Core schema 7 is clearly *Modus Ponens*, since that schema is, with other symbols, as follows:

$a \longrightarrow b$
a
____
Ergo b

On the other hand, Core schema 4 is similar to the version of *Modus Ponendo Tollens* indicated above, except that it has more conjuncts. With other symbols, it appears like this:

$\neg (a_1 \cdot \ldots \cdot a_n)$
a_i
____
Ergo $\neg (a_1 \ldots \cdot a_{i-1} \cdot a_{i+1} \cdot \ldots \cdot a_n)$ (Braine and O'Brien 1998b, 80)

Finally, another example is Braine and O'Brien's (1998b, 80) Core schema 2, which appears to be a version of *Modus Ponens* in which the antecedent is a disjunction. With other symbols, the schema appears like this:

$(a_i \text{ v} \ldots \text{v } a_n) \longrightarrow b$
a_i
____
Ergo b, where "v" stands for a disjunction.

Feeder schemata are quite interesting for this paper. Such schemata are only used when they can lead to a constative that in turn allows applying a Core schema. They are interesting because in Braine and O'Brien (1998b, 80) it is said that the two main Feeder schemata are exactly the two rules that, as indicated, Stoic logic could need but do not have. Those rules are obviously the *conjunction introduction rule* and the *conjunction elimination rule* in standard constatival calculus, and the numbers assigned to them by Braine and O'Brien (1998b, 80) are, respectively, 8 and 9. With other symbols, they look like these:

Schema 8:

a_1
a_2
$\ldots$

a_n

Ergo $a_1 \cdot a_2 \cdot \ldots \cdot a_n$

Schema 9:

$a_1 \cdot \ldots \cdot a_i \cdot \ldots \cdot a_n$

Ergo a_i

One might think that the Feeder schemata should be distinguished from the Core schemata because, if used whenever possible, the Feeder schemata lead to infinite derivations which do not describe what the human mind actually does. In this way, the idea of mental logic theory seems to be that, if they were Core schemata, then they could enable infinite deductions such as this one:

[1] a	(Premise)
[2] b	(Premise)
[3] a · b	(Schema 8; 1, 2)
[4] a	(Schema 9; 3)
[5] b	(Schema 9; 3)
[6] a · b	(Schema 8; 4, 5)
[7] a	(Schema 9; 6)
....	
[n] and so on.	

For this reason, it appears absolutely necessary to assume the restriction that they can only be used when, after that, a Core schema can be applied, too. Nonetheless, maybe this is not the only reason. As commented by Braine et al. (1998b, 149), experimental results and arguments such as those of Samuel Fillenbaum (1977) enable us to assume that people do not think they are actually reasoning or drawing a conclusion when they apply a schema such as Schemata 8 and 9 in Braine and O'Brien (1998b, 80). Individuals seem to think that they are only reorganizing or clarifying information that is already known. Therefore, it is obvious that the status of Feeder schemata cannot be the same as that of Core schemata.

This latter point is important because it can lead to the idea that the Stoics did not consider the need to have rules for introducing and eliminating conjunctions because, in their view, using such rules did not derive a conclusion or make an inference. They could think in a similar way as the general population and not note that those rules are necessary in deduction processes. If this thesis is right, the problem is solved. The Stoics thought that several particular conditionals could lead to a general conditional because a rule for linking such particular conditionals as conjuncts was not necessary. Doing that was not an inferential process, but an information reorganization process. Likewise, they thought that a particular conditional could be deduced from a general conditional because a rule for separating the conjuncts of the same conjunction was not also needed. Doing that was only an information reorganization process as well. So, it seems that Stoic logic was not only a particular constatival logic but it also enables us to work with general conditionals or general constatives, which are akin to universally quantified constatives, and to draw instantiations from them.

CONCLUSION

Only little information on conjunction in Stoic logic can be found in the ancient sources. For this reason, a speculative and theoretical work regarding that topic is needed. In this paper, I tried to do that work and, although my account may be improved, I think, that the current version is coherent with the philosophical and psychological literature on the sources.

Gould's (1970) theses allow us to assume that the Stoics built general conditionals from particular conditionals. On the other hand, O'Toole and Jennings' (2004) arguments enable us to suppose that they also drew particular conditionals from general conditionals.

The difficulty is that the conjunction is important in those two processes and the sources are not very clear on this logical operator. However, the mental logic theory has helped us in understanding the real function of it in Stoic logic. It can be assumed that the formal rules corresponding to conjunction are not Core schemata, but Feeder schemata, and that people can think that using them is not really reasoning. In this way, it can also be supposed that the Stoics considered conjunction introduction and conjunction elimination to be the obvious processes that do not need a specific rule to be applied.

NOTES

1. This paper is a result of Project N. I003011, "*Algoritmos adaptativos e inferencias lógicas con enunciados condicionales*," supported by the Directorate for Research of the University of Talca, Chile. As the main researcher of that Project, the author would like to thank that institution for its help in funding this paper.

2. I now adopt Rolando M. Gripaldo's (2010 and 2011) logical terminology and replace the word "proposition" with the word "constative." A constative is any utterance, spoken or written, that can be rendered true or false. Singular constatival logic refers to the traditional Aristotelian particular or universal statements and not to general constatives (or general conditionals), which include the general relations of "conjunction," "disjunction," "implication," and the like.

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Submitted: 20 March 2015; revised: 19 February 2016